# Elasticity and elastic constants

## e-content for B.Sc Physics (Honours)
## B.Sc Part-I

Paper-I

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**1.Introduction** Elasticity is the property by which a body resists change in its size or shape when an external force is acting on it and returns to the original state after the removal of the deforming force.

# 1.1 CLASSIFICATION OF ELASTIC MATERIALS

Elastic materials are classified into two types:

- Perfectly elastic
- Plastic

Materials which recover their original state after the removal of the deforming force are called *perfectly elastic materials*. Materials which do not recover their original state even after the removal of deforming force are called as *plastic materials***.** A material which does not undergo any relative displacement of its parts when an external force acts on it, however large it may be, is called a *perfectly rigid material***.**

No substance is perfectly elastic or perfectly plastic, since every substance tends to regain its equilibrium condition at least partially.

A quartz fibre which recovers most of its original state after a large deforming force is removed can be considered as perfectly elastic body. But a perfectly plastic body like putty can recover its original state only if the deforming force applied is very small.

## 1.2 Fundamental Definitions

### 1.2.1 Restoring Force

When an external force acts on a body to cause deformation, forces of reaction comes into play internally and tends to restore the body to its original condition. These internal forces are called *restoring forces*.

### 1.2.2 Stress

The restoring force or recovering force per unit area is called *stress*.

$$\text{Stress} = \frac{\text{Restoring force}}{\text{Area}} = \frac{F}{A}$$

Stress is expressed in $Nm^{-2}$ or Pascal.

### 1.2.3 Strain

The ratio of the change in dimension produced by an external force to its original dimension is known as *strain*. The nature of the strain depends on the nature of the deforming forces. Strain has no unit and dimension.

The ratio of change in length per unit length is known as *linear strain or longitudinal strain*. It is created by longitudinal stress.

When equal and opposite forces act tangentially along two opposite faces of a cube, a change in shape is produced. Such a strain is called *shearing strain*.

When an equal inward or outward force is applied normal to each face of a cube, a change in volume is produced. The ratio of the change in volume per unit volume is known as *volume strain*.

## 1.3 Hooke's Law

Robert Hooke, in 1679, proposed a relation between stress and strain. The maximum value of the stress within which a body completely regains its original condition of shape and size when the deforming forces are removed is known as the *elastic limit*.

Hooke's law states that within the elastic limit, the ratio of the stress to the strain is constant. This constant is called the *modulus of elasticity* of the material.

Stress $\alpha$ strain

Stress = a constant X Strain

$$\frac{Strain}{Stress} = \text{Constant}$$

The constant is a proportionality constant which is known as *modulus of elasticity*.

### 1.3.1 Types of Moduli of Elasticity

There are three modulus of elasticity:

- Young' modulus ($Y$)
- Bulk modulus ($K$)
- Rigidity modulus ($n$)

### 1.3.2 Young's Modulus of Elasticity (Y)

It is the ratio of longitudinal (tensile) stress to longitudinal strain. It is denoted by $Y$.

$$\text{Young's modulus } Y = \frac{\text{Longitudinal or linear stress}}{\text{Longitudinal or linear strain}} \quad (1.1)$$

Consider a wire of length $L$ and area of cross section $A$. One end of it is fixed while the other end is loaded as shown in Figure 2.1. Let $l$ be the extension produced in the wire for the given load.

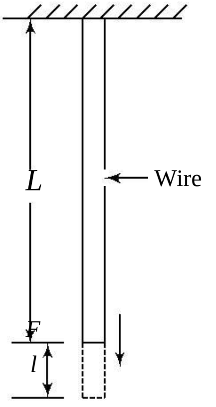



**Figure 2.1** *Young's modulus of elasticity*

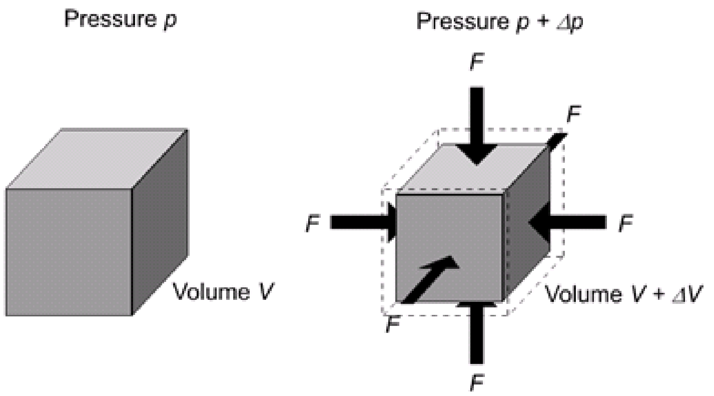
Longitudinal stress = $\frac{F}{A}$ $\text{Nm}^{-2}$

$$\frac{l}{L}$$

Young's modulus of elasticity

$$Y = \frac{F/A}{l/L} = \frac{FL}{Al} \text{Nm}^{-2} \text{ or Pascal} \qquad (1.2)$$

## 1.3.3 Bulk Modulus(K)



Suppose euqal forces act perpendicular to the six faces of a cube of volume V as shown in  Due to the action of these forces, let the decrease in volume be dV.

Now, Bulk stress = Force/Are = F/ A

Bulk Strain = change in volume / original volume = -dV/V

(The negative sign indicates that volume decreases.)

Bulk modulus of the material of the object is defined as the ratiobulk stress to bulk strain.

It is denoted by k

Bulk modulus =  Bulk stress / Bulk strain

K = -PV/dV

### 1.3.4 Rigidity Modulus

Consider a solid cube *ABCDEFGH* whose lower face *EFGH* is fixed. A tangential force *F* is applied on the upper face *ABCD* . Due to the application of the force, an equal and opposite force comes into play on the lower fixed face *EFGH*. These two forces form a couple, which makes the layers parallel to the two faces to move one over the other. Thus, the point *A* shifts to *A*¢, *B* to *B*¢, *C* to *C*¢ and *D* to *D*¢ (Figure 2.3).

The line joining the two faces turn through an angle *q*. The face *ABCD* is said to be sheared through an angle *q*.

The shearing stress is the tangential force per unit area of the face *ABCD*.

$$\text{Rigidity modulus } n = \frac{\text{Shearing stress}}{\text{Shearing strain}} \quad (1.5)$$

$$\text{Sharing stress} = \frac{F}{A} \text{ Nm}^{-2}$$

$$\text{Sharing strain} = \tan q = \frac{l}{L}$$

As *q* is very small, tan *q* can be written as *q*.

$$\text{Shearing strain} = q = \frac{l}{L}$$

$$\text{Rigidity modulus of elasticity } n = \frac{F / A}{q} = \frac{F}{Aq} \text{ Nm}^{-2} \text{ or Pascal}$$

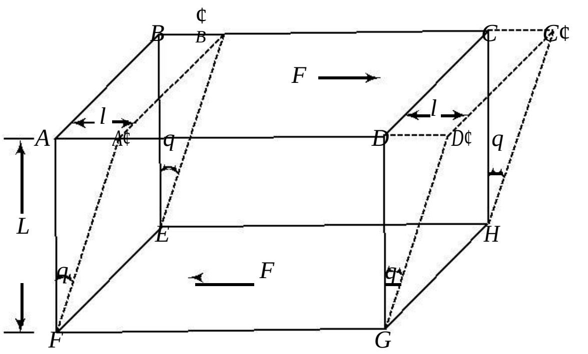



**Figure 1.3** *Rigidity modulus of elasticity*

## 1.4 Stress-Strain Diagram

Consider a body subjected to a uniformly increasing stress which results in a change in its dimension. The elastic behavior of a material is studied from the graph plotted between different stresses applied to the material and the corresponding strain produced in it. This graph, as shown in Figure 1.7, is called stress-strain curve.

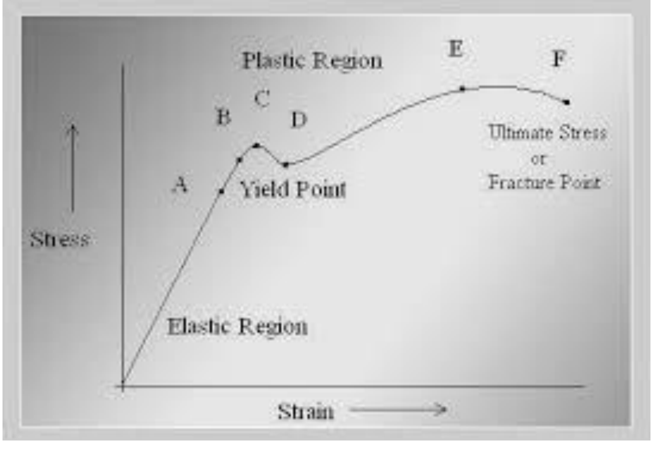



**Figure 1.7** *Stress-strain curve*

Stress strain curve has different regions and points. These regions and points are:

(i). Proportional limit
(ii). Elastic limit
(iii). Yield point
(iv). Ultimate stress point
(v). Fracture or breaking point.

**(i). Proportional Limit:** It is the region in the strain curve which obeys hookes law i.e. within elastic limit the stress is directly proportion to the strain produced in the material. In this limit the ratio of stress with strain gives us proportionality constant known as young's modulus. The point OA in the graph is called the proportional limit.

**(ii). Elastic Limit:** It is the point in the graph upto which the material returns to its original position when the load acting on it is completely removed. Beyond this limit the material cannot return to its original position and a plastic deformation starts to appear in it. The point A is the Elastic limit in the graph.

**(iii). Yield Point or Yield Stress Point:** Yield point in a stress strain diagram is defined as the point at which the material starts to deform plastically. After the yield point is passed there is permanent deformation develops in the material and which is not reversible. There are two yield points and it is upper yield point and lower yield point. The stress corresponding to the yield point is called yield point stress. The point B is the upper yield stress point and C is the lower yield stress point.

**(iv) Ultimate Stress Point:** It is the point corresponding to the maximum stress that a material can handle before failure. It is the maximum strength point of the material that can handle the maximum load. Beyond this point the failure takes place. Point D in the graph is the ultimate stress point.

**(v). Fracture or Breaking Point:** It is the point in the stress strain curve at which the failure of the material takes place. The fracture or breaking of material takes place at this point. The point e is the breaking point in the graph.

## Uses of stress –strain diagram

1. It is used to measure the elastic strength yield strength and tensile strength of metals
2. It is used to estimate the working stress and safety factor of an engineering material.
3. This diagram is also used to identify the ductile and brittle materials.
4. The area under the curve in the elastic region gives the energy required to deform it elastically. The area under the curve upto Ultimate Tensile strength(UTS) gives the energy required to deform it plastically.

### 1.5 Factors Affecting Elasticity

The following are some of the important factors which affect the elastic properties of solids.

- **Stress:** The action of large constant stress or the repeated number of cycles of stresses acting on a body affect the elasticity of the body gradually. Considering this fact, the working stress on an engineering material is kept well below its ultimate tensile strength.
- **Temperature:** The elasticity of material decreases with the increase of temperature. A carbon filament which is highly elastic at normal temperatures becomes plastic when it is at high temperatures. Lead is not a good elastic material but at low temperatures it becomes a very good elastic material. Creep resistance is a property by which the material can withstand

its elastic property without fracture at high temperatures and during quick loading. Dispersion hardened materials and coarse hardened materials have better creep resistance at high temperature. Hence they can withstand their elastic properties even at high temperatures.

- **Impurities:** The elastic property of a material may increase or decrease due to the addition of impurities. If we add carbon in minute quantities to molten iron, the elastic properties of iron are increased enormously. But when the carbon content is more than 1% in iron, then the strength of iron decreases. Similarly, the addition of potassium in gold increases the elastic properties of gold.

  If any addition of impurity atoms distorts the lattice structure of the base metal, then the elastic property of the base metal decreases. This kind of impurity atoms generally have different atomic radii

  and electronic structures from those of the base metal atoms and therefore act as centres of distortion which decrease the elastic properties of the base metal.
- **Crystalline nature:** For a given metal, the modulus of elasticity is more when it is in single crystal form. But in the polycrystalline state, its modulus of elasticity is comparatively small, while its mechanical properties like ductility, malleability, machinability, etc., increase. Hence, polycrystalline form of metals is used in most of the engineering works.
- **Heat treatment and metal processing:** A grain of elastic material con-sists of many small interlocking crystals. Various heat treatment pro-cesses are adopted to get the desired physical and mechanical proper-ties through the changes in micro constituents of the material. Anneal-ing (heating and then slow cooling) is adopted to increase softness and ductility in the materials. But it decreases the tensile strength and yield point of the material due to formation of large crystal grains.

  Hammering and rolling are metal processing techniques to make thin plates and sheets. They break the grains into fine grains and increase its elastic properties. Metals with fine grains are stronger than metals with large or coarse grains. However for high temperature applications, materials with large grains are used since they have high creep resistance.